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# COMBINED PROBLEM OF SLEW MANEUVER CONTROL AND VIBRATION SUPPRESSION

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#### **ABSTRACT**

In this paper, the combined problem of slew maneuver control and vibration suppression of NASA Spacecraft Control Laboratory Experiment (SCOLE) is considered. The coupling between the rigid body modes and the flexible modes together with the effect of the control forces on the flexible antenna is discussed. The nonlinearities in the equations are studied in terms of slew maneuver angular velocities.

#### INTRODUCTION

In this paper, the analytics for the combined problem of slew maneuver and vibration suppression are developed. It is assumed that the slew maneuver is performed by applying moments on the rigid shuttle and the vibration suppression is achieved by means of forces on the flexible antenna and the reflector. The slew maneuver is considered to be an arbitrary maneuver about any given axis [16]. The effect of slew maneuver angular velocity on flexible modes is studied by examining the spectral norm of the matrix term associated with the coupling between the rigid-body modes and the flexible modes. Also, the kinematic nonlinearities are further analyzed in terms of the matrix spectral norm variation of the corresponding term with respect to slew maneuver angular velocity.

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#### **ANALYTICS**

The slew maneuver is defined as

- $\lambda$  Axis about which the slew maneuver is performed.
- $\xi$  The slew Angle
- $\underline{\omega}$  The angular velocity of the orbiter in the inertial frame.

The four Euler parameters can be defined as

$$\epsilon_{1} = \lambda_{1} \sin \frac{\xi}{2}$$

$$\epsilon_{2} = \lambda_{2} \sin \frac{\xi}{2}$$

$$\epsilon_{3} = \lambda_{3} \sin \frac{\xi}{2}$$

$$\epsilon_{4} = \cos \frac{\xi}{2}$$
(1)

The four Euler parameters can be related to the angular velocity components of the rigid assembly as

$$\begin{bmatrix} \dot{\epsilon}_1 \\ \dot{\epsilon}_2 \\ \dot{\epsilon}_3 \\ \dot{\epsilon}_4 \end{bmatrix} = \begin{bmatrix} \epsilon_1 & \epsilon_4 & -\epsilon_3 & \epsilon_2 \\ \epsilon_2 & \epsilon_3 & \epsilon_4 & -\epsilon_1 \\ \epsilon_3 & -\epsilon_2 & \epsilon_1 & \epsilon_4 \\ \epsilon_4 & -\epsilon_1 & -\epsilon_2 & -\epsilon_3 \end{bmatrix} \begin{bmatrix} 0 \\ \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix}$$
 (2)

The slewing maneuver can be given in terms of the following equations [16]

$$I_o \dot{\underline{\omega}} + A_2 \ddot{\underline{g}} = \underline{G}(t) + \underline{N}_2(\underline{\omega}) \tag{3}$$

$$A_2^T \underline{\dot{\omega}} + A_3 \underline{\ddot{q}} + B\underline{\dot{q}} + K\underline{q} = Q(t)$$
 (4)

where,

 $\underline{G}(t)$  is the net moment applied about the mass center of the orbiter and is

given by the following equation (figs. 1 & 2)

$$\underline{G}(t) = \underline{G}_o(t) + (\underline{r} + \underline{a}) \times \underline{F}_2 \tag{5}$$

Also, Q(t) represents the generalized force vector which is given by the following equation

$$Q(t) = \begin{bmatrix} \sum_{j=1}^{m} (Q_{jx_1}(t) + Q_{jy_1}(t)) + Q_{x_1} + Q_{y_1} + Q_{\psi_1} \\ \sum_{j=1}^{m} (Q_{jx_2}(t) + Q_{jy_2}(t)) + Q_{x_2} + Q_{y_2} + Q_{\psi_2} \\ \dots \\ \dots \\ \sum_{j=1}^{m} (Q_{jx_l}(t) + Q_{jy_l}(t)) + Q_{x_l} + Q_{y_l} + Q_{\psi_l} \\ \dots \\ \dots \\ \dots \end{bmatrix}$$
(6)

where, the generalized force components are given as

$$Q_{jx_{i}} = \int_{0}^{L} F_{jx}(z,t) \delta(z-z_{j}) \phi_{xi}(z) dz$$
 (7)

$$Q_{jx_{i}} = \int_{0}^{L} F_{jx}(z,t) \delta(z-z_{j}) \phi_{xi}(z) dz$$

$$Q_{jy_{i}} = \int_{0}^{L} F_{jy}(z,t) \delta(z-z_{j}) \phi_{yi}(z) dz$$
(8)

and

$$Q_{j\psi_l}(t) = 0 (9)$$

Here,  $F_{jx}(z,t)$  is the x component of the concentrated force applied at location j on the flexible antenna and  $F_{jy}$  is the y component of that force.

Also,

$$Q_{xi}(t) = F_{2x}(t)\phi_{xi}(L)$$

$$Q_{vi}(t) = F_{2v}(t)\phi_{vi}(L)$$
 (10)

$$Q_{\psi i}(t) = M_{\psi}(t)\phi_{\psi i}(L)$$

Here,  $\underline{F}_2$  is the force applied at the reflector C. G. Thus,

$$M_{\psi}(t) = F_{2x}r_{y} + F_{2y}r_{x} + M_{2\psi}$$
 (11)

The location of reflector C. G. is given by coordinates  $(r_x, r_y)$  and  $M_{2\psi}$  represents the external moment applied at the reflector C. G. Also, the nonlinearities  $\underline{N}_2$  can be expressed in terms of pure rigid body kinematic nonlinearity and the nonlinear coupling term between the rigid-body modes and the flexible modes.

$$\underline{N}_{2} = A_{4} \left( \underline{\omega}, \underline{\theta} \right) + A_{5} \left( \underline{\omega}, \underline{\theta} \right) \underline{\dot{q}}$$
 (12)

### (a) Slew Maneuver

If only a slew maneuver is to be considered, then  $Q(t) \equiv \underline{0}$  and  $F_2 \equiv \underline{0}$ , and only moments are applied at the orbiter C. G. However, the angular velocity vector  $\underline{\omega}$ , is nonzero during the maneuver and the flexible modes will be excited. This effect of coupling between the rigid-body modes and flexible modes can be obtained by evaluating  $A_5$  which depends on the angular velocity vector. In figure 3, using the matrix spectral norm as a measure, the coupling effect is studied as a function of slew angular velocity. The first ten flexible modes are considered for this analysis. The kinematic nonlinearity is also obtained in terms of matrix spectral norm as a function of  $\underline{\omega}$ . This analysis can be utilized in the linearization of the slew maneuver dynamical equations. An example of this is shown in figure 4 which is a single plane slew maneuver. In this case, it is almost a linear relationship in terms of a single angular velocity component.

## (b) Slew Maneuver Control and Vibration Suppression

If it is desired to design control systems for the simultaneous task of slew maneuver control and vibration suppression, then equations (3)-(11) should be used. It can be seen that vibration control forces also affect the slew maneuver dynamics through control moment coupling terms.

Thus, these equations would suggest that in order to achieve control efficiency and to minimize the line of sight error in minimum time, it may be necessary to synthesize control systems for the combined problem of slew maneuver and vibration suppression.

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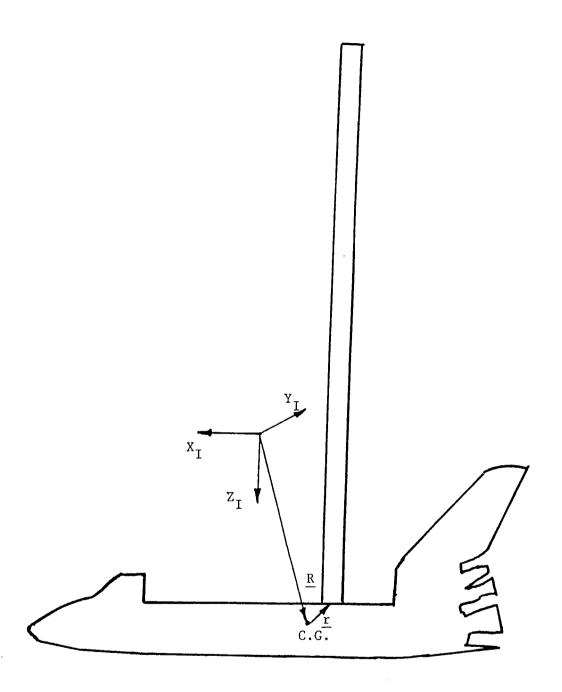


Figure 1- Position Vectors in Inertial Frame

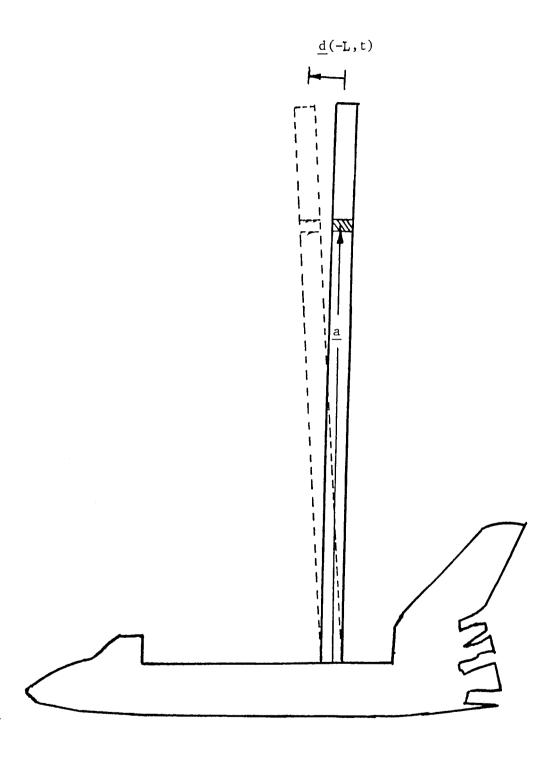


Figure 2- Vectors in Body-fixed Frame

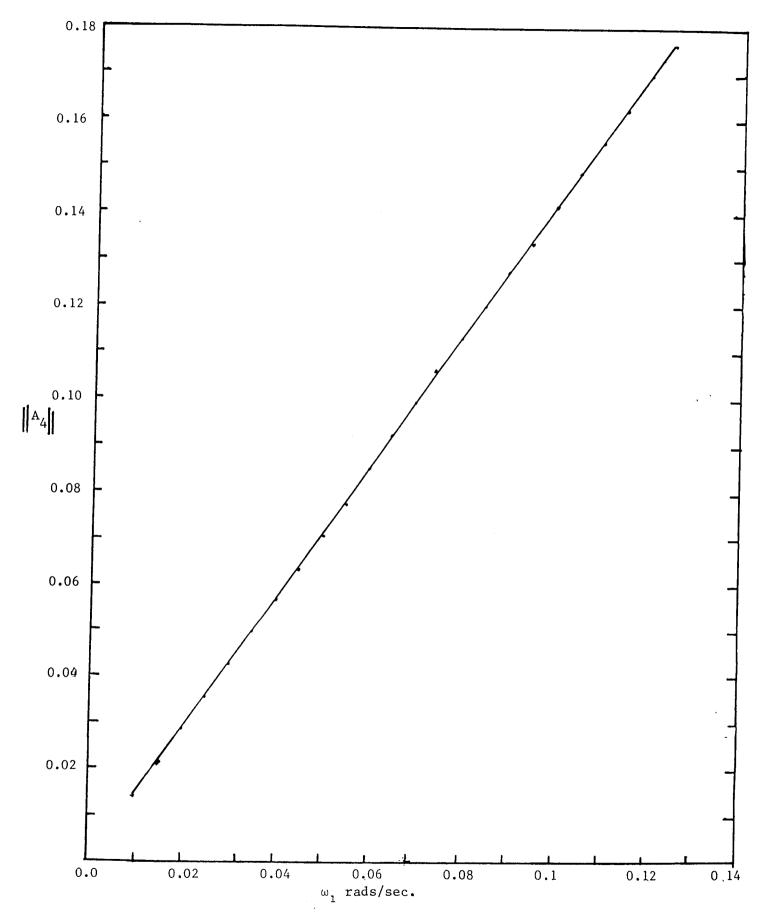


Figure 3-  $\|A_4\|$  vs  $\omega_1$  (  $\omega_2 = \omega_3 = 0$  )

